

CLASS
9th

Maths

EXERCISE

NO:- 4.1

U#04)

EXERCISE NO: 4.1

Basic Concepts:

* Factor:

A number that divides the given number exactly without a remainder

In case of numbers: $2 \times 3 = 6$

↓ ↓
factor factor

In case of expression:

$$(x+3)(x+1) = x^2 + 4x + 3$$

↓ ↓
factor factor

* Factorization:

(Factorization is the process of finding the factors.)

Example: $2y+6$

$$= 2(y+3)$$

↓ ↓
factor factor

$ab+ac$

$$ab+ac = a(b+c)$$

↓ ↓
factor factor

Basic formulas:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a-b)(a+b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$(a+b)^3 = a^3 + 3ab(a+b) + b^3$

$(a-b)^3 = a^3 - 3ab(a-b) - b^3$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

* How to find mid term: *

$$\begin{array}{ccc} \text{1st term} & \text{Mid term} & \text{Last term} \\ \uparrow & \uparrow & \uparrow \\ x^2 & + 7x & + 10 \end{array}$$

* سب سے پہلے (Last term) کو (1st term) سے

سے (Multiply) کرتے ہیں

* اس کے بعد LCM لیتے ہیں

* :: کچھ ایسے دو (pairs) بناتے ہیں جن

کو (x) کرنے سے آخری ٹرم بیک اور (+) یا

(-) کرنے سے درمیان والی

* :: کچھ دو (pairs) بنا کر (common) لیتے ہیں

$$x^2 + 7x + 10$$

Solve:

$$x^2 + 7x + 10$$

$$= x^2 + 5x + 2x + 10$$

$$= x(x+5) + 2(x+5)$$

$$= (x+5)(x+2)$$

$$x^2 \times 10 = 10x^2$$

LCM

$$\begin{array}{r} 2 \overline{) 10} \\ 4 \\ \hline 6 \\ \hline 10 \end{array}$$

$$\begin{array}{r} 5 \overline{) 5} \\ 5 \\ \hline 0 \end{array}$$

$$(x+2x)(x+5) = 10x^2$$

$$+ 2x + 5x = +7x$$

*

$$y^2 - 12y - 64$$

factorization

$$\begin{aligned}
 &= y^2 - 16y + 4y - 64 \\
 &= y(y-16) + 4(y-16) \\
 &= (y-16)(y+4)
 \end{aligned}$$

$$y^2x - 64 = -64y^2$$

LCM:

$$\begin{array}{r}
 16 \left(\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \end{array} \right) \\
 4 \left(\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2 \\ \hline 2 & 1 \end{array} \right)
 \end{array}$$

$$\begin{aligned}
 (-16y)(4y) &= -12y^2 \\
 -16yx \cdot 4y &= -64y^2
 \end{aligned}$$

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Exercise # 4.1

Factorize the following polynomials.

Q NO: 1

$$2x^2y^3 - 6x^2y^2 + 2xy^3$$

Solve:

$$\begin{aligned} 2x^2y^3 - 6x^2y^2 + 2xy^3 \\ = 2xy^2(1xy - 3x + 1y) \end{aligned}$$

Q NO: 2

$$3nx - 3x - 3ny + 3y$$

Solve:

$$\begin{aligned} 3nx - 3x - 3ny + 3y \\ = 3[nx - x - ny + y] \\ = 3[x(n-1) - y(n-1)] \\ = 3[(n-1)(x-y)] \end{aligned}$$

OR 2nd Method

$$\begin{aligned} = 3nx - 3x - 3ny + 3y \\ = 3x(n-1) - 3y(n-1) \\ = (n-1)(3x-3y) \\ = (n-1)3(x-y) \\ = 3(n-1)(x-y) \end{aligned}$$

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Q NO: 3

$$18x^4 + 108x^2y^2 + 162y^4$$

Solve

$$18x^4 + 108x^2y^2 + 162y^4$$

$$= 18[x^4 + 6x^2y^2 + 9y^4]$$

$$\because (a+b)^2 = a^2 + 2ab + b^2$$

$$= 18[(x^2)^2 + 2(x^2)(3y^2) + (3y^2)^2]$$

$$= 18[(x^2 + 3y^2)^2]$$

Q NO: 4

$$(k+2)^2 - 8(k+2) + 16$$

Solve:

$$(k+2)^2 - 8(k+2) + 16$$

$$= (k+2)^2 - 2(k+2)(4) + (4)^2$$

$$\because (a-b)^2 = a^2 - 2ab + b^2$$

$$= (k+2-4)^2$$

$$= (k-2)^2$$

Q NO: 5

$$9x^2 + 4 - 169y^2 - 12x$$

Solve:

$$9x^2 + 4 - 169y^2 - 12x$$

$$= 9x^2 + 4 - 12x - 169y^2$$

Re-arrange

$$= [9x^2 - 12x + 4] - 169y^2$$

$$\because (a-b)^2 = a^2 - 2ab + b^2$$

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$$= [(3x)^2 - 2(3x)(2) + (2)^2] - (13y)^2$$

$$= [(3x-2)^2] - (13y)^2$$

$$\therefore a^2 - b^2 = (a-b)(a+b)$$

$$= [(3x-2) + 13y] [(3x-2) - 13y]$$

$$= [3x - 2 + 13y] [3x - 2 - 13y]$$

$$= [3x + 13y - 2] [3x - 13y - 2]$$

Q NO: 6

$$(x^2 - 1)(y - 1) - (y + 3)(x^2 - 1)$$

Solve

$$(x^2 - 1)(y + 1) - (y + 3)(x^2 - 1)$$

$$= (x^2 - 1) [(y + 1) - (y + 3)]$$

$$= (x^2 - 1) [y + 1 - y - 3]$$

$$= (x^2 - 1) [+1 - 3]$$

$$= (x^2 - 1) (-2)$$

$$= -2(x^2 - 1)$$

$$\therefore a^2 - b^2 = (a-b)(a+b)$$

$$= -2 [(x)^2 - (1)^2]$$

$$= -2 [(x-1)(x+1)]$$

$$= -2 (x-1)(x+1)$$

Q NO: 7

$$x^2 - 6ax + 9a^2 - 16b^2$$

Solve:

$$x^2 - 6ax + 9a^2 - 16b^2$$

$$[x^2 - 6ax + 9a^2] - 16b^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

$$[(x)^2 - 2(x)(3a) + (3a)^2] - (4b)^2$$

$$[(x-3a)^2] - (4b)^2$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$[(x-3a) + 4b] [(x-3a) - 4b]$$

$$[x - 3a + 4b] [x - 3a - 4b]$$

Q NO: 8

$$1 - x^2 - 2xy - y^2$$

Solve

$$1 - x^2 - 2xy - y^2$$

$$1 - [x^2 + 2xy + y^2]$$

$$\because a^2 + 2ab + b^2 = (a+b)^2$$

$$(1)^2 - [(x)^2 + 2(x)(y) + (y)^2]$$

$$(1)^2 - [(x+y)^2]$$

$$\because a^2 - b^2 = (a-b)(a+b)$$

$$[1 + (x+y)] [1 - (x+y)]$$

$$[1 + x + y] [1 - x - y]$$

Q NO: 9

Solution:-

Given that

$$(x+y-2c)(x+2c+y)$$

$$(x+y-2c)(x+y+2c)$$

$$\therefore (a-b)(a+b) = a^2 - b^2$$

$$= (x+y)^2 - (2c)^2$$

$$\therefore (a+b)^2 = a^2 + 2ab + b^2$$

$$= [(x)^2 + (y)^2 + 2(x)(y)] - (2c)^2$$

$$= [x^2 + y^2 + 2xy] - 4c^2$$

$$= x^2 + y^2 + 2xy - 4c^2$$

Q NO: -10

Solution:-

Given expression:

$$x^2 + 4y^2 - z^2 + 4xy$$

Re-arrange:

$$x^2 + 4xy + 4y^2 - z^2$$

$$[x^2 + 4xy + 4y^2] - (z)^2$$

$$\therefore a^2 + 2ab + b^2 = (a+b)^2$$

$$= [(x)^2 + 2(x)(2y) + (2y)^2] - (z)^2$$

$$= [(x+2y)^2] - (z)^2$$

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Q NO: 11

$$a) (2y^2 - 3y - 27) = (y+3) (\quad)$$

Solution:

$$2y^2 - 3y - 27$$

factorization:

$$= 2y^2 - 9y + 6y - 27$$

$$= y(2y-9) + 3(2y-9)$$

$$= (2y-9)(y+3)$$

$(2y-9)$ is missing factor.

$$b) (5x^2 + 12x - 9) = (\quad) (x+3)$$

Solution:

$$5x^2 + 12x - 9$$

factorization:

$$= 5x^2 + 15x - 3x - 9$$

$$= 5x(x+3) - 3(x+3)$$

$$= (x+3)(5x-3)$$

$(5x-3)$ is missing factor.

-45

3 | 45

3 | 15

5 | 5

1

(+15x)(-3)

Q NO: 12

$$x^4 + 4m^4$$

Solve:-

$$x^4 + 4m^4$$

$$= (x^2)^2 + (2m^2)^2 + 2(x^2)(2m^2) - (2)(x^2)(2m^2)$$

$$\therefore a^2 + 2ab + b^2 = (a+b)^2$$

$$= (x^2 + 2m^2)^2 - 4m^2x^2$$

$$= (x^2 + 2m^2)^2 - (2mx)^2$$

$$\therefore (a-b)(a+b) = a^2 - b^2$$

$$= [(x^2 + 2m^2) - 2mx] [(x^2 + 2m^2) + 2mx]$$

$$= [x^2 + 2m^2 - 2mx] [x^2 + 2m^2 + 2mx]$$

Q NO: 13

Solution:

$$m^4 + m^2 + 1$$

Re-arrange

$$m^4 + 1 + m^2$$

$$= (m^2)^2 + (1)^2 + 2(m^2)(1) - 2(m^2)(1) + m^2$$

$$\therefore a^2 + 2ab + b^2 = (a+b)^2$$

$$= (m^2 + 1)^2 - 2m^2 + m^2$$

$$= (m^2 + 1)^2 - m^2$$

$$= (m^2 + 1)^2 - (m)^2$$

$$\therefore a^2 - b^2 = (a-b)(a+b)$$

$$= [(m^2 + 1) + m] [(m^2 + 1) - m]$$

$$= [m^2 + 1 + m] [m^2 + 1 - m]$$

$$= [m^2 + m + 1] [m^2 - m + 1]$$

Q NO: 14

Solution:

$$\begin{aligned} & -3x^4 - 21x^3 + 24x^2 \\ & = -3x^2 [x^2 + 7x - 8] \\ & = -3x^2 [x^2 + 8x - 1x - 8] \\ & = -3x^2 [x(x+8) - 1(x+8)] \\ & = -3x^2 [(x+8)(x-1)] \\ & = -3x^2 (x+8)(x-1) \end{aligned}$$

Q NO: 15

Solution:

$$\begin{aligned} & x^8 + x^4 + 1 \\ & \text{Re-arrange} \\ & = x^8 + 1 + x^4 \\ & = (x^4)^2 + (1)^2 + 2(x^4)(1) - 2(x^4)(1) + x^4 \\ & \because a^2 + 2ab + b^2 \\ & = (x^4 + 1)^2 - 2x^4 + x^4 \\ & = (x^4 + 1)^2 - x^4 \\ & = (x^4 + 1)^2 - (x^2)^2 \\ & \because a^2 - b^2 = (a-b)(a+b) \\ & = [(x^4 + 1) + x^2] [(x^4 + 1) - x^2] \\ & = [x^4 + 1 + x^2] [x^4 + 1 - x^2] \\ & = [(x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2] \\ & \quad [x^4 + 1 - x^2] \end{aligned}$$

$$\begin{aligned}
&= [(x^2+1)^2 - 2x^2 + x^2] [x^2+1-x^2] \\
&= [(x^2+1)^2 - x^2] [x^2-x^2+1] \\
&= [(x^2+1)^2 - (x)^2] [x^2-x+1] \\
&= [(x^2+1)+x][(x^2+1)-x] [x^2-x+1] \\
&= [x^2+1+x] [x^2+1-x] [x^2-x+1] \\
&= [x^2+x+1] [x^2-x+1] [x^2-x+1]
\end{aligned}$$

Q NO: 16

Solve:

$$\begin{aligned}
&4x^4 + 256y^4 \\
&= 4[x^4 + 64y^4] \\
&= 4[(x^2)^2 + (8y^2)^2 + 2(x^2)(8y^2) - 2(x^2)(8y^2)] \\
&= 4[(x^2+8y^2)^2 - 16x^2y^2] \\
&= 4[(x^2+8y^2)^2 - (4xy)^2] \\
&= 4[(x^2+8y^2) + 4xy] [(x^2+8y^2) - 4xy] \\
&= 4[x^2+8y^2+4xy] [x^2+8y^2-4xy]
\end{aligned}$$

Q NO: 17

Solve:

$$\begin{aligned}
 & 12 - 7x + x^2 \\
 & \quad \text{Re-arrange} \\
 & = x^2 - 7x + 12 \\
 & \quad \text{factorization} \\
 & = x^2 - 4x - 3x + 12 \\
 & = x(x-4) - 3(x-4) \\
 & = (x-4)(x-3)
 \end{aligned}$$

Q NO: 18

Solve:

$$x^2 - 9x + 8$$

factorization:-

$$x^2 - 8x - 1x + 8$$

$$x(x-8) - 1(x-8)$$

$$(x-8)(x-1)$$

Q NO: 19

Solve:

$$10z^2 - 29z + 10$$

factorization:

$$= 10z^2 - 25z - 4z + 10$$

$$= 5z(2z-5) - 2(2z-5)$$

$$= (2z-5)(5z-2)$$

Q NO: 20

Solve:

$$-3y^2 + 13y - 4$$

$$= -[3y^2 - 13y + 4]$$

$$= -[3y^2 - 12y - 1y + 4]$$

$$= -[3y(y-4) - 1(y-4)]$$

$$= -[(y-4)(3y-1)]$$

Q NO: 21

Solve:

$$x^2 - 21x + 90$$

factorization

$$= x^2 - 6x - 15x + 90$$

$$= x(x-6) - 15(x-6)$$

$$= (x-6)(x-15)$$

Q NO: 22

Solve:

$$x^2 + x - 2$$

factorization

_ | _ | 30

$$= x^2 + 2x - x - 2$$

$$= x(x+2) - 1(x+2)$$

$$= (x+2)(x-1)$$

Q No: 23

Solve:

$$3x^2 + 11x + 6$$

factorization

$$= 3x^2 + 9x + 2x + 6$$

$$= 3x(x+3) + 2(x+3)$$

$$= (x+3)(3x+2)$$

Q No: 24

Solve:

$$2x^2 - 5xy - 3y^2$$

factorization

$$= 2x^2 - 6xy + xy - 3y^2$$

$$= 2x(x-3y) + y(x-3y)$$

$$= (x-3y)(2x+y)$$

Q No: 25

Solve:

$$8 + 6x - 5x^2$$

Factorizations-

$$= 8 - 4x + 10x - 5x^2$$

$$= 4(2-x) + 5x(2-x)$$

$$= (2-x)(4+5x)$$

Q No: 26

Solve:

$$6 - 7x - 5x^2$$

Factorizations

$$= 6 + 3x - 10x - 5x^2$$

$$= 3(2+x) - 5x(2+x)$$

$$= (3-5x)(2+x)$$

OR Second Method:

$$6 - 7x - 5x^2$$

$$- 5x^2 - 7x + 6$$

$$- 5x^2 - 10x + 3x + 6$$

$$- 5x(x+2) + 3(x+2)$$

$$(x+2)(3-5x)$$

Q No: 27

Solve:

$$2a^2 - 4a - 6$$

factorization:

$$= 2a^2 - 6a + 2a - 6$$

$$= 2a(a-3) + 2(a-3)$$

$$= (a-3)(2a+2)$$

$$= (a-3)2(a+1)$$

$$= 2(a-3)(a+1)$$

$$\begin{array}{r} 2a^2x-6 \\ -12a^2 \end{array}$$

$$\begin{array}{r} 2 \overline{) 12} \\ \underline{2 } \\ 6 \\ \underline{6 } \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ \underline{-(6a) } \\ 6a \\ \underline{-(6a) } \\ 0 \end{array} = -4a$$

Method:



$$2a^2 - 4a - 6$$

$$= 2[a^2 - 2a - 3]$$

$$= 2[a^2 - 3a + a - 3]$$

$$= 2[a(a-3) + 1(a-3)]$$

$$= 2[(a-3)(a+1)]$$

$$= 2(a-3)(a+1)$$

Q No: 28

Solve:-

$$u^4 - 13u^2 + 36$$

factorization:

$$u^4 - 9u^2 - 4u^2 + 36$$

$$\begin{aligned}
& u^2(u^2-9) - 4(u^2-9) \\
&= (u^2-9)(u^2-4) \\
&= [(u)^2 - (3)^2] [(u)^2 - (2)^2] \\
&= [(u+3)(u-3)] [(u-2)(u+2)] \\
&= (u+3)(u-3)(u-2)(u+2)
\end{aligned}$$

Q No:- 29

Solve

$$y^4 - 12y^2 - 64$$

factorization

$$\begin{aligned}
& y^4 - 16y^2 + 4y^2 - 64 \\
&= y^2(y^2 - 16) + 4(y^2 - 16) \\
&= (y^2 - 16)(y^2 + 4) \\
&= [(y)^2 - (4)^2] (y^2 + 4) \\
&= [(y+4)(y-4)] (y^2 + 4) \\
&= (y+4)(y-4)(y^2 + 4)
\end{aligned}$$