Exercise 3.5

1. If s varies directly as u^2 and inversely as v and $s = 7$ when $u = 3$, $v = 2$. Find the value of s when $u = 6$ and $v = 10$.

$S \propto u^2$

$S \propto \frac{1}{v}$

$S \propto \frac{u^2}{v}$

$S = k\frac{u^2}{v}$ —— (i)

$S = 7, U = 3, v = 2$

$7 = \frac{k(3)^2}{2}$

$7 \times 2 = 9k$

$14 = 9k$

$$\frac{14}{9} = K$$

To find S

$u = 6$

$v = 10$

$K = \frac{14}{9}$

Put above values in eqn (1)

$$S = \frac{Ku^2}{v}$$

$$= \frac{14(6)^2}{9 \times 10}$$

$$= \frac{14 \times \cancel{36}^{2}}{\cancel{90}_{5}} \text{ (Cutting by 18)}$$

$$= \frac{14 \times 2}{5} \Rightarrow \frac{28}{5} = S$$

2. If w varies jointly as x, y^2 and z and $w = 5$ when $x = 2$, $y = 3$, $z = 10$. Find w when $x = 4$, $y = 7$ and $z = 3$.

$$w \propto x y^2 z$$

$$w = k x y^2 z \quad \text{— (i)}$$

$$w = 5, \; x = 2, \; y = 3, \; z = 10$$

Put above mentioned values in (i)

$$5 = K(2)(3)^2(10)$$

$$5 = K\,20(9)$$

$$5 = K\,180$$

$$\frac{5}{180} = K$$

$$36$$

$$K = \frac{1}{36}$$

To Find w

$$x = 4 \, , \; y = 7 , z = 3 , K = \frac{1}{36}$$

Put in equ (i)

$$w = K x y^2 z$$

$$w = \frac{1}{36}(4)(7)^2(3)$$

$$w = \frac{\cancel{12}^{1}(49)}{\cancel{36}_{3}}$$

$$\boxed{w = \frac{49}{3}}$$

3. If y varies directly as x^3 and inversely as z^2 and t, and $y = 16$ when $x = 4$, $z = 2$, $t = 3$. Find the value of y when $x = 2$, $z = 3$ and $t = 4$.

$$y \propto x^3$$

$$y \propto \frac{1}{z^2 t}$$

$$y \propto \frac{x^3}{z^2 t}$$

$$y = \frac{K x^3}{z^2 t} \quad \text{— (i)}$$

Put $x = 4$, $y = 16$, $z = 2$, $t = 3$

in eqn (i)

$$16 = \frac{K(4)^3}{(2)^2(3)}$$

$$16 = \frac{K\,64}{4 \times 3}$$

$$16 = \frac{k64}{12}$$

$$16 \times 12 = 64k$$

$$\frac{16 \times 12}{64} = k$$

$$\frac{192}{64} = k$$

$$\frac{3}{1}$$

$$\boxed{k = 3}$$

To Find y

Put $x=2$, $z=3$, $t=4$ and $k=3$ in (i)

$$y = \frac{kx^3}{z^2 t}$$

$$y = \frac{(3)(2)^3}{(3)^2(4)}$$

$$y = \frac{3 \times 8}{9 \times 4}$$

$$y = \frac{\cancel{24}^{2}}{\cancel{36}_{3}} \quad \text{(cutting by 12)}$$

$$y = \frac{2}{3}$$

4. If u varies directly as x^2 and inversely as the product yz^3, and $u = 2$ when $x = 8$, $y = 7$, $z = 2$. Find the value of u when $x = 6$, $y = 3$, $z = 2$.

$$u \propto x^2$$

$$u \propto \frac{1}{yz^3}$$

$$u \propto \frac{x^2}{yz^3}$$

$$u = \frac{Kx^2}{yz^3} \quad \text{——— (i)}$$

Put $u = 2$, $x = 8$, $y = 7$, $z = 2$ in eqn (i)

$$2 = \frac{K(8)^2}{(7)(2)^3}$$

$$2 = \frac{64K}{(7)(8)}$$

$$(2)(8)(7) = 64K$$

$$\frac{(2)(8)(7)}{64} = K$$

$$\frac{16(7)}{64_4} = K$$

$$\boxed{K = \frac{7}{4}}$$

To Find v

put $x = 6, y = 3, z = 2, K = \frac{7}{4}$

in equ (i)

$$v = \frac{Kx^2}{y^3 z^2}$$

$$v = \frac{7(6)^2}{4(3)^3(2)^3}$$

$$U = \frac{7 \times 36^{3}}{12 \times 8}$$

$$\boxed{U = \frac{21}{8}}$$

5. If v varies directly as the product xy^3 and inversely as z^2 and $v = 27$ when $x = 7$, $y = 6$, $z = 7$. Find the value of v when $x = 6$, $y = 2$, $z = 3$.

$$v \propto xy^3$$

$$v \propto \frac{1}{z^2}$$

$$v \propto \frac{xy^3}{z^2}$$

$$v = \frac{k\,xy^3}{z^2} \quad \text{——— (i)}$$

Put $x = 7$, $y = 6$, $z = 7$

$v = 27$ in eqn (i)

$$27 = \frac{k\,7\,(6)^3}{(7)^2}$$

$$27 = \frac{k\,7\,(216)}{49\,7}$$

$$27 \times 7 = 216K$$

$$\frac{27 \times 7}{216} = K$$

$$8$$

$$\boxed{\frac{7}{8} = K}$$

To Find v

Put $x = 6,\ y = 2,\ z = 3,\ K = \frac{7}{8}$

in eqn (i)

$$V = \frac{Kxy^3}{z^2}$$

$$V = \frac{7(6)(2)^3}{8(3)^2}$$

$$V = \frac{7 \times \cancel{6} \times \cancel{8}^{2}}{\cancel{8} \times \cancel{4}\ 3}$$

$$V = \frac{14}{3}$$

If w varies inversely as the cube of u, and $w = 5$ when $u = 3$. Find w when $u = 6$.

K-Method

$$w \propto \frac{1}{v^3}$$

$$w = K\frac{1}{v^3}$$

$$K = v^3 w \quad \text{—— (i)}$$

$$K = (3)^3(5)$$

$$K = 27 \times 5$$

$$\boxed{K = 135}$$

To Find w

Put $v = 6$, $K = 135$ in eq(i)

$$K = v^3 w$$

$$\frac{K}{v^3} = w$$

$$\frac{135}{(6)^3} = \omega$$

$$\frac{135}{216} = \omega$$

$$\frac{5}{8} = \omega$$