$x^2+y^2+z^2=r^2$ (1)

$x^2+y^2+z^2=r^2$ Ans

Exercise 6.5
Textbook Page (136)

Q1: From a point at a distance of 20 m from a tree, angle of elevation of top of a tree is 30°. Find height of the tree.

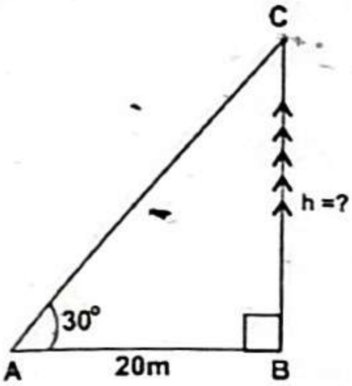
Solution:

Let h be the height of the tree

$\theta = 30^0$ (angle of elevation)

$AB = 20m$ (distance from the tree)

In ΔABC



$$\tan\theta = \frac{prep}{base}$$

$$\tan 30^0 = \frac{h}{20}$$

$$0.577 = \frac{h}{20}$$

$$0.577 \times 20 = h$$

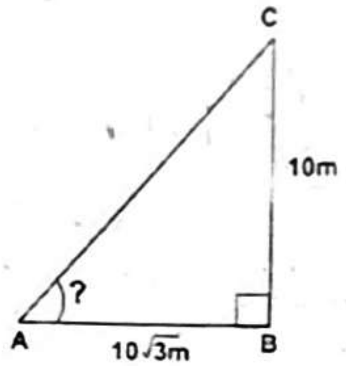
$$\boxed{h = 11.55m}$$

$\therefore$ height of the tree = 11.55m **Ans**

Q2: The length of shadow of 10 m high pole is $10\sqrt{3}$ m. Find angle of elevation of the sun.

Let 'θ' be the angle of elevation.

In ΔABC



$$\tan\theta = \frac{prep}{base}$$

$$\tan\theta = \frac{10}{10\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

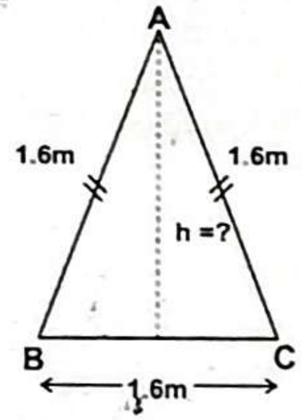
$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\boxed{\theta = 30^0}$$

Angle of elevation = 30^0 **Ans**

Q3: The window of a room is in the shape of an equilateral triangle. The length of each side is 1.6m. Find height of the window.

Sol: Consider the figure



Let height of the window is 'h'

We know that

Each angle of an equilateralΔ is 60^0, so

$\angle A = \angle B = \angle C = 60^0$

In ΔABC

$$\sin\angle B = \frac{prep}{hyp}$$

$$\sin 60^0 = \frac{h}{1.6}$$

$$0.866 = \frac{h}{1.6}$$

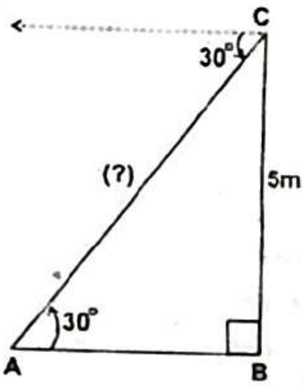
$$1.6 \times 0.866 = h$$

$$\boxed{h = 1.39m}$$

∴ height of the window is 1.39m Ans

Q4: The height of a slide in a children park is 5m. A girl standing at the top of the slide observes that angle of depression of its bottom is 30°. Find length of the slide.

Solution:



Let 'x' be the length of the slide.

Height of the slide = 5m

Angle of elevation

= angle of depression

Angle of elevation = 30^0

$\Rightarrow \theta = 30^0$

In ΔABC

$$\sin\theta = \frac{prep}{hyp}$$

$$\sin 30^0 = \frac{5}{x}$$

$$0.5 = \frac{5}{x}$$

$$0.5x = 5 \Rightarrow x = \frac{5}{0.5}$$

$$\boxed{x = 10m}$$

so, length of the slide is 10m **Ans**

Q5: Two pillars of equal height stand on either side of a roadway which is 120m wide. At a point on the road between pillars, the angles of elevation of the pillars are 60° and 30°. Find height of each pillar and position of the point.

Solution:

Let's denote the height of the pillars as (h) Since the pillars are of equal height We need to find (h). Let the position of the point on the road be (x) meters from the base of the pillars with the 60° angle of elevation, and (120 – x) meters from the base of pillar with the 30° angle of elevation.

For the 60° angle of elevation.

$$\text{Tan } 60^o = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}x \rightarrow (1)$$

For the 30° angle of elevation.

$$\tan 30^o = \frac{h}{120 - x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{120 - x} \rightarrow (2)$$

Put equation (1) in equation (2)

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{120 - x}$$

$$(\sqrt{3})(\sqrt{3}x) = 1(120 - x)$$

$$(\sqrt{3})^2 x = 120 - x$$

$$3x + x = 120$$

$$4x = 120$$

$$x = \frac{120}{4}$$

$$\boxed{x = 30m}$$

Put x = 30 in (1)

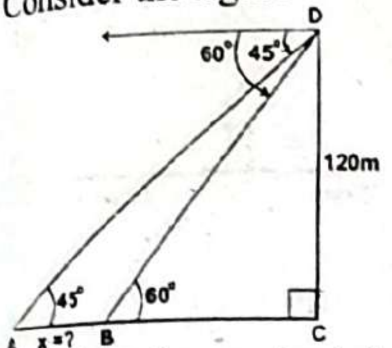
$h=\sqrt{3}x$
$h=\sqrt{3}(30)$
$h=1.732\times 30$
$\boxed{h=52m}$

So, the height of each pillar is 52m and the position of the point is 30m.

Q6: From the top of a tower of height 120m, angles of depression of two boats on same side of tower at water level are 60^o and 45^o. Find distance between the boats.

Solution:

Consider the figure



Let 'x' be the required distance between two boats.

In ΔABC

$\tan 60^0 = \frac{120}{BC}$

$1.732 = \frac{120}{BC}$

$1.732\, BC = 120$

$BC = \frac{120}{1.732}$

$\boxed{BC = 69.28m}$

In ΔACD;

$\tan 45^0 = \frac{120}{AC}$

$1 = \frac{120}{AB+BC}$

$1 = \frac{120}{x+69.28}$

$x+69.28=120$

$x=120-69.28$

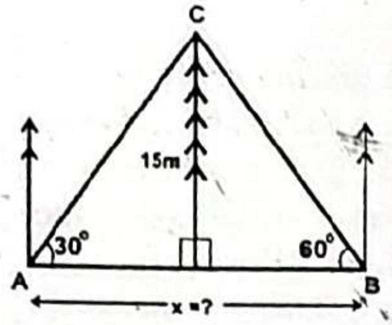
$\boxed{x=50.72m}$

So, the distance between two boats is 50.72m **Ans.**

Q7: Two men on opposite sides of a tree are in line with it. They observe that angles of elevation of top of tree are 30^o and 60^o. The height of tree is 15 m. Find distance between the men.

Solution:

Consider the figure.



Let x be the required distance between the two men.

Height of the tree = CD = 15m

$\angle A = 30^0, \angle B = 60^0$

In ΔACD

$\tan 30^0 = \frac{15}{AD}$

$0.577 = \frac{15}{AD}$

$0.577 AD = 15$

$AD = \frac{15}{0.577}$

$\boxed{AD = 26m}$

In ΔBCD

$\tan 60^0 = \frac{15}{BD}$

$1.732 = \frac{15}{BD}$

$1.732 BD = 15$

$BD = \frac{15}{1.732}$

$\boxed{BD = 8.7m}$

$x = AD + BD$

$x = 26m + 8.7m$

$\boxed{x = 34.7m}$

So, the distance b/w the two men is 34.66m **Ans**

Q8: From the top of a tree, the angles of elevation and depression of the top and bottom of a building are 30^0 and 45^0 respectively. If height of tree is 12m, find height of building and distance between tree and building.

Solution:

i. Let the height of the building be h meters.
ii. Let the distance between the tree and the building be x meters.

From the top of the tree, the angle of elevation of the top of the building is 30^0, so

$$\tan 30^0 = \frac{h-12}{x}$$

$$0.577 = \frac{h-12}{x}$$

$$0.577x = h-12$$

$$h = 0.577x + 12 \rightarrow (1)$$

From the top of the tree, the angle of depression of the bottom of the building is 45^0, so

$$\tan 45^0 = \frac{12}{x}$$

$$1 = \frac{12}{x}$$

$$\boxed{x = 12m}$$

Put x = 12 in eq (1)

h = 0.577x + 12

h = 0.577(12) + 12

h = 6.92 + 12

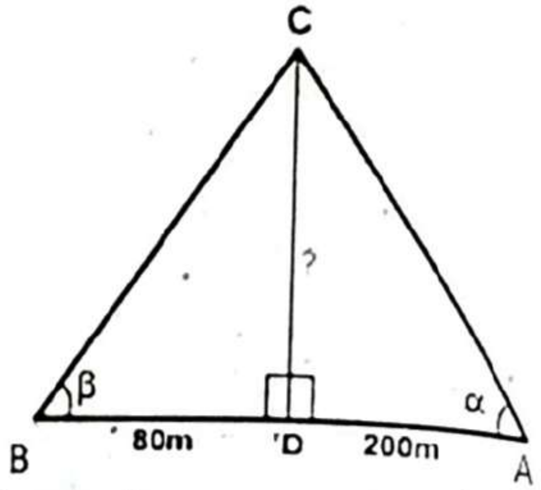
$\boxed{h = 18.92m}$

So, the height of the building is 18.92 meters, and the distance between the tree and the building is 12 meters. **Ans**

Q9: From a point A on ground at a distance of 200m, angle of elevation of top of a tower is α. There is another point B 80 m nearer to the tower. The angle of elevation of top of tower from B is β. If $\tan \alpha = \frac{2}{5}$, find height of tower an value of β.

Solution:

Consider the figure:



Required:

Height of tower =?

β =?

Let height of the tower = x

BD = 80m, AD = 200m, also

$$\tan \alpha = \frac{2}{5}$$

In ΔACD

$$\tan\alpha = \frac{CD}{AD}$$

$$\frac{2}{5} = \frac{x}{200}$$

$$5x = 400$$

$$x = \frac{400}{5}$$

$$\boxed{x = 80m}$$

In ΔBCD

$$\tan\beta = \frac{CD}{BD}$$

$$\tan\beta = \frac{80}{80}$$

$$\tan\beta = 1$$

$$\beta = \tan^{-1}(1)$$

$$\boxed{\beta = 45^0}$$

∴ Height of the tower = 80m and $\beta = 45^{0}$

Ans

Q10: A boat moving away from a light house 206.6 m high, takes 120 seconds to change the angle of elevation of top of light house from 60^{o} to 45^{o}. Find the speed of boat.

Solution:

First, we need to find the distance b/w the boat at the lighthouse when the angle of elevation is 60^{o} and 45^{o}.

Using the tangent function

$$\tan 60^{o} = \frac{206.6}{x}$$

$$x = \frac{206.6}{\tan 60^{o}}$$

$$\boxed{x = 118.6m}$$

$$\tan 45^{o} = \frac{206.6}{y}$$

$$y = \frac{206.6}{\tan 45^{o}}$$

$$\boxed{y = 206.6m}$$

Where x and y are the distances.

Now, we can find the distance travelled by the boat

Distance = y − x

Distance = 206.6 − 118.6

Distance = 88m

Since time = 120 seconds

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Speed} = \frac{88m}{120 \text{ seconds}}$$

Speed = 0.733 m/s

The speed of the boat is 0.733 meters per second.